METHODS FOR PERFORMANCE EVALUATION OF PRECISION POSITIONING SYSTEMS: POINT REPEATABILITY

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INTRODUCTION

Precision motion systems are used in a variety of industry applications varying from, but not limited to, metrology, nano-machining, photonics, laser marking, and manufacturing. The selection and use of a specific precision motion system is dependent on the ability of the system to meet the performance required for the application. With the advancement of application requirements, including increasing levels of precision and introduction of new performance metrics, both the stage technology and testing methodology have evolved to be able to meet and prove these performance requirements. As a result, many motion system manufacturers have developed their own testing definitions and procedures where the uncertainty limits of existing standardized tests restrict their applicability or where there is no test procedure to cover a performance metric [1,2,3,4]. This can lead to confusion and mis-information in industry as test procedures and results, which are used for comparison and qualification, are not based on a consistent set of principles.

Repeatability of linear and angular stages is a performance metric where existing standard test uncertainty is larger than current repeatability performance capabilities and existing repeatability definitions do not clearly address multi-dimensional repeatability required for many applications [5]. This paper presents a updated proposal for Point Repeatability terminology and test methodology for characterizing repeatability of a functional point (attached to a stage) in three-dimensional space with appropriate test uncertainty ratios.

This proposal is presented in conjunction with the preliminary work conducted by a working group of Technical Committee TC52 (ASME B5/TC52) tasked with developing and drafting a new standard for precision motion systems.

REPEATABILITY DEFINITIONS

Per ASME B5.54-2005, repeatability is "a measure of the ability of a machine to sequentially position a tool with respect to a work-piece under similar conditions." [6] A similar definition is applied in ISO 230-2:2006(E) [7].

Existing Definition: (Planar) Repeatability

Per test procedures and calculations in ASME B5.54-2005, motion system manufacturers specify repeatability as a single number representing the variation in linear displacement. The one-dimensional nature of the test means that each test position measured is conceptually defining the location of a plane along the axis, see Figures 1 and 2. The test does not tell us any information about the pitch, yaw, or roll of this plane and we do not know where our actual test point is located horizontally or vertically on the plane. We only know one piece of information, where the plane is along an assumed perfectly straight axis. This one-dimensional, widely accepted repeatability can also be called planar repeatability [5].

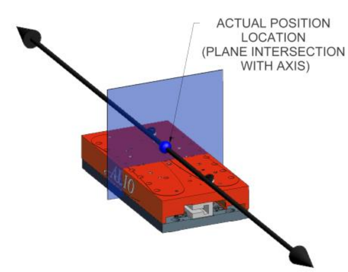


FIGURE 1. The intersection of a plane (fixed to a moving carriage) along an assumed perfect axis is a positional data point used in traditional repeatability testing.

Referencing the same top view as shown in Figure 2, an additional error source, straightness error, is added and shown in Figure 3. Two possible examples of the actual repeatability in a 2D plane are shown. On each of the plots the planar repeatability (as measured per ASME B5.54) would be characterized to be equivalent between the two situations, see the "measured repeatability" callout. Clearly the "unmeasured" straightness repeatability perpendicular to the axis of travel varies greatly between the two situations.

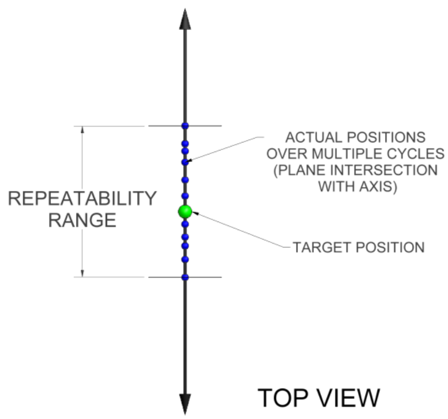


FIGURE 2. Traditional repeatability test method and theory results in a range of points along an axis, which characterizes one-dimensional linear repeatability.

While current standards include methods to test multiple repeatability error sources (i.e. linear, straightness, flatness, pitch, yaw, and roll repeatability), they do not address the following:

- there is no terminology framework for specifying repeatability as a multi-dimensional performance metric,
- there is no method combining individual one-dimensional repeatability data into a representation of repeatability in full three-dimensional space, and
- the uncertainty limitations of existing test methods make characterization at the nanometer level difficult, if not impossible.

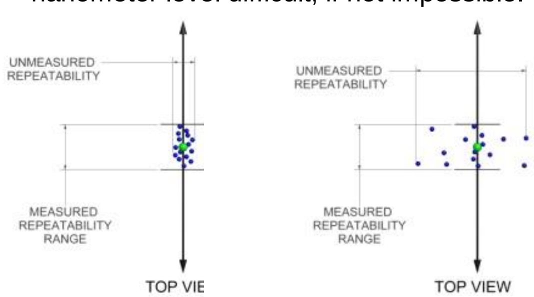
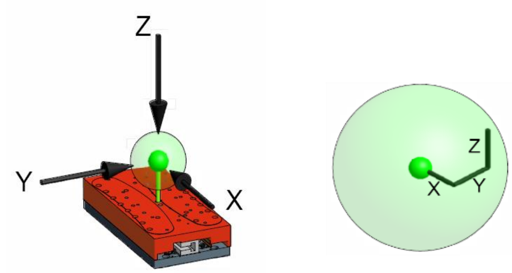


FIGURE 3. Two independent representations of repeatability of a linear axis in a 2D plane.

Defining Point Repeatability

At the nanometer level, each axis of a motion system has errors in six degrees of freedom, and these error sources may be statistically significant and should be accounted for in a repeatability test method [4]. Applications that require nanometer order performance generally require it in two- or three-dimensional space, not in only one dimension. As a result, there is a void between application requirements for performance in three-dimensional space and existing test methods that characterize errors in one dimension (or degrees of freedom).

In order to account for all error sources, we will conceptually look at a functional test point fixed to a moving carriage of an axis and examine how that point returns to a target position in space over multiple cycles. By examining a point (in contrast to a plane), we can quantify how that point moves in three-dimensional space given stage errors in any of the six degrees of freedom: X, Y, Z, pitch, yaw, or roll, see Figure 4(a). ***Point Repeatability is a measure of the ability of a motion system to sequentially position a functional point fixed to a stage with respect to a fixed three-dimensional reference frame.*** Point repeatability is specified on a per system basis. A system could be a single axis or multi-axis system but currently only single axis procedures are addressed in this document.



(a) (b)

FIGURE 4. A representation of (a) X, Y, and Z repeatability components of a functional point attached to a motion carriage and (b) the spherical tolerance zone that will encapsulate all positions of the test point.

Equation 0.a and 0.b compare the error sources accounted for in (traditional) planar repeatability and point repeatability concepts. Clearly planar repeatability is only a function of an error in one linear degrees of freedom, $\epsilon_{x,}$ and repeatability components in the other degrees of freedom are neglected. The point repeatability concept captures the impact of all error sources.

$Planar\ Repeatability = f(\epsilon_x)$ (Eqn. 0-a)

$Point\ Repeatabilty = f(\epsilon_{x,}\epsilon_{y,}\epsilon_{z,}\epsilon_{\beta,}\epsilon_{\alpha,}\epsilon_{\gamma})$ (Eqn. 0-b)

where the repeatability error sources of a moving carriage are,

- ϵ_x - repeatability error in linear X DOF
- ϵ_y - repeatability error in linear Y DOF
- ϵ_z - repeatability error in linear Z DOF
- ϵ_β - repeatability error in angular pitch DOF
- ϵ_α - repeatability error in angular yaw DOF
- ϵ_γ - repeatability error of angular roll DOF

Over many cycles, the test point locations will create a three-dimensional distribution of points. While the shape of the distribution will vary from stage to stage or manufacturer to manufacturer, a sphere is the common shape that will fully encapsulate all points in the three-dimensional repeatability distribution. This sphere is defined as the ***Point Repeatability Spherical Tolerance Zone, which is the spherical tolerance zone around a target position that contains all measured test point locations over multiple bidirectional cycles of a point repeatability test.***

In order to characterize the point repeatability we will characterize the X, Y, and Z components of the point repeatability distribution and use the component distributions to calculate the spherical distribution that will contain all points, see Figure 4(b).

We also defined two other terms that will be used throughout this document. ***Functional Point is a test point rigidly attached to a motion system's moving carriage***. There are an infinite number of possible functional points for every stage and in general the test point should be defined to coincide with the process location specified by the end user. ***The Target Position is a position to which the moving functional point is programmed to move.*** This definition is the same as specified in ASME B5.54-2005.

Prior Work

The Point Repeatability concept has been previously used to document repeatability in three-dimensional space. This proposal is motivated at standardizing definitions and procedures of this prior work. Most notably it is referenced in ASME B5.54-2005 in reference to tool change repeatability where three-dimensional repeatability is measured with a three sensor nest, see Figure 5.

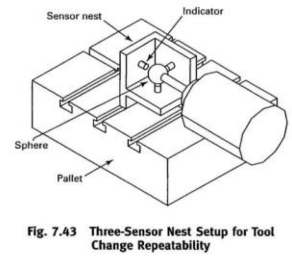


FIGURE 5. Three-sensor nest setup for tool change repeatability. Figure 7.43 in ASME B5.54-2005. [7]

TEST METHODOLOGY

The following methodology outlines the process for analyzing X, Y and Z repeatability components of a functional point and using that data to calculate the Point Repeatability.

Define Functional Point and Target Positions

First the testing locations must be defined. This includes the defining of two points: the functional point fixed to the motion system carriage and the target position along the axis of travel to which this test point will be sequentially positioned during test measurement. Test locations or test points must be defined per procedures specified in ASME B5.54. Specifically, "Measurement intervals shall be no larger than 25mm for axes of 250mm or less . For longer axes the intervals shall be no more than 1/10 of the axis length." [7]

The functional point location should be agreed upon with the customer or user to coincide with the process/application. It should be noted that different functional point locations will result in different quantitative results based on the magnitude of pitch, yaw, and roll repeatability error components, see Figure 6.

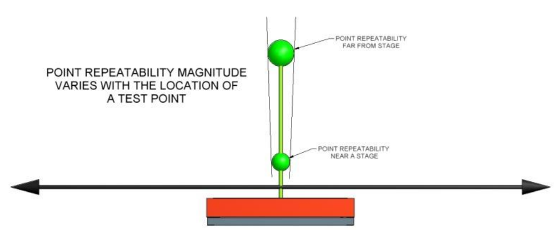


FIGURE 6. A visual representation of the impact pitch repeatability will have on actual point repeatability magnitude recorded.

Test Setup

This test methodology is mainly focused on calculating Point Repeatability given the X, Y, and Z repeatability component datasets of a functional point. It should be noted that the test setups used to collect this data are not strictly controlled by this methodology. Thus the procedures documented in ASME 5.54-2005 to collect linear, straightness, and flatness repeatability are valid procedures to collect the required datasets. Likewise this methodology also allows other sensor arrangements such as the three sensor nest of capacitive gauges, as shown in Figure 5.

The main requirement of the motion during testing is that the target position be approached bidirectionally, unless it is agreed upon with the user that the application only requires uni-directional performance assessment. This motion cycle may resemble a bidirectional pendulum test or the full travel bidirectional cycle as specified in Figure 7.11 of ASME B5.54-2005, see Figure 7.

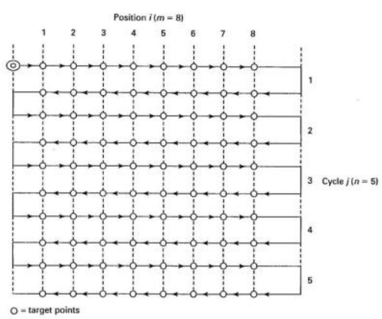


Fig. 7.11 Standard Test Cycle

FIGURE 7. A standard full travel bidirectional repeatability test cycle - Figure 7.11 from ASME B5.54-2005 shown for reference.[7]

Data Analysis

For each target point there will be three arrays.

$Ex_{i,j}$ X directional repeatability data
$Ey_{i,j}$ Y directional repeatability data
$Ez_{i,j}$ Z directional repeatability data

where for all presented equations,

i = test point along the axis of travel
j = data point (1,2,3,...) at each test point
$n = j_{max} = 20$

The following equations parallel equations 7-2 to 7-18 in ASME B5.54-2005 but are differentiated in that unidirectional performance is not calculated. Uni-directional performance is addressed by customization of the motion cycle performed during data selection.

The mean position is defined as follows.

$$\bar{E}x_i \uparrow\downarrow = \frac{1}{n}\sum_{j=1}^{n} Ex_{i,j} \qquad \text{(Eqn. 1-a)}$$
$$\bar{E}y_i \uparrow\downarrow = \frac{1}{n}\sum_{j=1}^{n} Ey_{i,j} \qquad \text{(Eqn. 1-b)}$$
$$\bar{E}z_i \uparrow\downarrow = \frac{1}{n}\sum_{j=1}^{n} Ez_{i,j} \qquad \text{(Eqn. 1-c)}$$

The mean is assumed to be the target position as the data models a normal distribution. The error of each data point from the mean is:

$$E'x_{i,j} = Ex_{i,j} - \bar{E}x_i \uparrow\downarrow \qquad \text{(Eqn. 2-a)}$$
$$E'y_{i,j} = Ey_{i,j} - \bar{E}y_i \uparrow\downarrow \qquad \text{(Eqn. 2-b)}$$
$$E'z_{i,j} = Ez_{i,j} - \bar{E}z_i \uparrow\downarrow \qquad \text{(Eqn. 2-c)}$$

For these adjusted data sets the mean position deviation is zero.

$$\bar{E}'x_i \uparrow\downarrow = 0 \qquad \text{(Eqn. 3-a)}$$
$$\bar{E}'y_i \uparrow\downarrow = 0 \qquad \text{(Eqn. 3-b)}$$
$$\bar{E}'z_i \uparrow\downarrow = 0 \qquad \text{(Eqn. 3-c)}$$

The estimator of the bidirectional standard uncertainty of the X, Y, and Z components at the test point is:

$$Sx_i \uparrow\downarrow = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(E'x_{i,j} - \bar{E}'x_i \uparrow\downarrow)^2} \qquad \text{(Eqn. 4-a)}$$
$$Sy_i \uparrow\downarrow = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(E'y_{i,j} - \bar{E}'y_i \uparrow\downarrow)^2} \qquad \text{(Eqn. 4-b)}$$
$$Sz_i \uparrow\downarrow = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(E'z_{i,j} - \bar{E}'z_i \uparrow\downarrow)^2} \qquad \text{(Eqn. 4-c)}$$

The repeatability X, Y, and Z components, at a test point are the deviation from the target position derived from the expanded uncertainty of positional deviations at a position using coverage factor of two.

$$Rx_i \uparrow\downarrow = \pm\ 2Sx_i\ \uparrow\downarrow \qquad \text{(Eqn. 5-a)}$$
$$Ry_i \uparrow\downarrow = \pm\ 2Sy_i\ \uparrow\downarrow \qquad \text{(Eqn. 5-b)}$$
$$Rz_i \uparrow\downarrow = \pm\ 2Sz_i\ \uparrow\downarrow \qquad \text{(Eqn. 5-c)}$$

The point repeatability spherical tolerance zone radius, PR_i, for test point, *i*, is the combination of the X, Y, and Z point repeatability components.

$$PR_i = \sqrt{(Rx_i \uparrow\downarrow)^2 + (Ry_i \uparrow\downarrow)^2 + (Rz_i \uparrow\downarrow)^2} \qquad \text{(Eqn. 6)}$$

Please note that the author acknowledges that the above equation is not statistically true and is currently re-evaluating how to calculate the radius distance of the sphere.

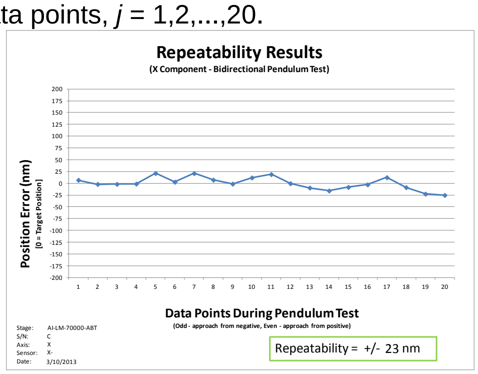
When a motion system is analyzed the point repeatability of the system, PR_{system}, is the maximum value of the individual test point repeatability values, PR_i.

$$PR_{system} = max.[PR_i] \qquad \text{(Eqn. 7)}$$

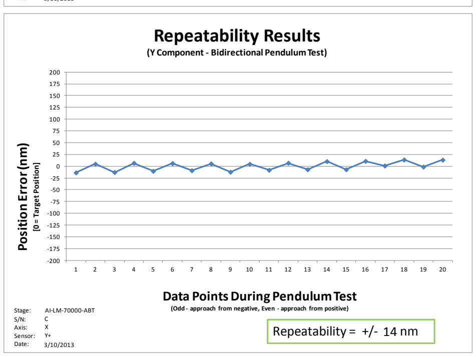
EXAMPLE TEST DATA

The following example presents data for a 700mm linear travel air bearing stage. For this setup, target points were defined every 100mm of travel for a total of 8 target points and a bidirectional pendulum test motion was used at each target position. A sensor nest similar to Figure 5 was used to collect X, Y, and Z component data for each target point. The functional point was defined at the front centered location (50mm height) of the mounting carriage (offset from the center of the carriage) to match the end user process location.

Below are the repeatability charts and tables for the X, Y, and Z components of the Point Repeatability for point i = 5. Each chart shows 20 data points, j = 1,2,...,20.



(a)



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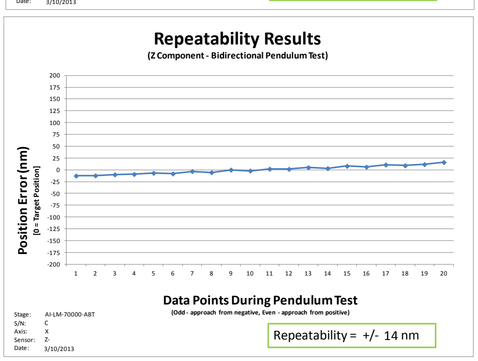

(c)

FIGURE 7. The (a) X, (b) Y, and (c) Z components of the raw repeatability for target point, i = 5.

TABLE 1. Tabulated data for target point, i = 5.

Target Point, i = 5			
Tolerance Zone Component	X Component	Y Component	Z Component
units	nm	nm	nm
Variable (Equation 2)	$E'x_{(5,j)}$	$E'y_{(5,j)}$	$E'z_{(5,j)}$
j = 1	6.7	-13.7	-12.8
j = 2	-2.1	4.6	-12.4
j = 3	-1.9	-13.0	-10.3
j = 4	-1.3	6.6	-9.5
j = 5	21.1	-10.5	-6.9
j = 6	2.7	6.1	-8.3
j = 7	20.9	-9.2	-3.3
j = 8	7.0	5.3	-5.7
j = 9	-1.4	-12.4	-0.2
j = 10	11.7	4.5	-2.2
j = 11	18.8	-8.3	1.8
j = 12	-0.6	6.7	1.4
j = 13	-9.7	-7.1	4.8
j = 14	-16.0	10.2	3.0
j = 15	-8.4	-7.1	7.9
j = 16	-2.9	10.7	5.9
j = 17	12.2	0.9	10.6
j = 18	-8.9	13.7	8.9
j = 19	-22.7	-1.4	11.4
j = 20	-25.3	13.3	15.8
Standard Deviation			
Units	nm	nm	nm
Variable (Equation 4)	Sx_4 ↑↓	Sy_4 ↑↓	Sz_4 ↑↓
i = 5	13.2	9.3	8.5

TABLE 2. The X, Y, and Z components and Point Repeatability radius for all target points is shown in Table 2.

2 Sigma Component Repeatability (+/-)			
Tolerance Zone Component	X Component	Y Component	Z Component
units	nm	nm	nm
Variable (Equation 5)	Rx_i ↑↓	Ry_i ↑↓	Rz_i ↑↓
i = 1 (@ -350mm)	25.7	19.1	7.0
i = 2 (@ -250mm)	25.7	11.3	5.1
i = 3 (@ -150mm)	32.1	13.9	5.0
i = 4 (@ -50mm)	29.0	26.2	9.5
i = 5 (@ +50mm)	26.4	18.7	17.1
i = 6 (@ +150mm)	30.0	4.6	12.8
i = 7 (@ +250mm)	35.8	13.4	13.6
i = 8 (@ +350mm)	24.6	12.6	3.6
Point Repeatability of all Test Points			
Units	nm	nm	nm
Variable (Equation 6)	PR_i		
i = 1 (@ -350mm)	32.8		
i = 2 (@ -250mm)	28.5		
i = 3 (@ -150mm)	35.3		
i = 4 (@ -50mm)	40.2		
i = 5 (@ +50mm)	36.6		
i = 6 (@ +150mm)	32.9		
i = 7 (@ +250mm)	40.6		
i = 8 (@ +350mm)	27.9		
Linear Point Repeatability PR_{system} (Equation 7)			
+/- 39.4 nm (+/- 11.0 nm Uncertainty)			

UNCERTAINTY ANALYSIS

This new test procedure using the three sensor nest results in deviations from existing uncertainty calculations as documented in ISO 230-9:2005(E) Informative Annex C. It is fully expected these equations will be updated once input from additional experts is incorporated.

The main differences in these uncertainty calculations compared to the Informative Annex C theory result from the following concepts.

- Point repeatability does not divide collected data into unidirectional approach sub-sets.
- Setup and misalignment error is introduced between each test position and is included.
- Uncertainty due to thermal effects is zero.

- The accuracy of the measurement values is significan thus device error is included.

All uncertainty contributors used below are used as defined in ISO 230-9:2005(E).

Proposed Point Repeatability Uncertainty Calculations

The X, Y, or Z individual component repeatability is +/- two times the standard deviation. The uncertainty is in accordance with equation C13 from Annex C of ISO230-9:2005(E) [8].

$$u(Rc) = 2 * \sqrt{\frac{1}{n-1}} * u_{EVE} \quad \text{(Eqn U-1)}$$

$$U(Rc) = 2 * u(Rc) \quad \text{(Eqn U-2)}$$

where,

$U(Rc)$ is the uncertainty of component repeatability data, *k* = 2;

n is the number of runs, n = 10.

The uncertainty of the X, Y, and Z data sets when accounting for accuracy and setup errors between data sets is as follows.

$$u(Rs) = \sqrt{u_{Rc}^2 + u_{DEVICE}^2 + u_{MISALIGNMENT}^2 + u_{SETUP}^2} \quad \text{(Eqn U-3)}$$

$$U(Rs) = 2 * u(Rs) \quad \text{(Eqn U-4)}$$

where,

$U(Rs)$ is the uncertainty of component repeatability, *k* = 2;

Uncertainties for X, Y, and Z components are assumed equal, thus $u_x = u_y = u_z = u(Rs)$.

$$u(PR) = \sqrt{3 * u(Rs)^2} \quad \text{(Eqn U-5)}$$

$$U(PR) = 2 * u(PR) \quad \text{(Eqn U-6)}$$

Example Uncertainty Calculations

The resulting uncertainty of the example presented is +/- 11.0 nm. (Type A analysis was used, JCGM 100:2008 [9].)

TABLE 3. Uncertainty contributors for the provided example.

Uncertainty Contributors (Type A)		
Device Error	u_{DEVICE}	1.9 nm
Misalignment Error	$u_{MISALIGNMENT}$	0.7 nm
Setup Error	u_{SETUP}	1.9 nm
Environmental Error	u_{EVE}	1.2 nm
Thermal Error	$u_{TEMPERAATURE}$	0.0 nm

FUTURE WORK

The authors have presented the proposed Point Repeatability terminology and test methods as a starting point for discussion with industry and academia. There are several areas where future work will lead to modifications, discussion, or correction of the principles put forth above.

- Clarification of spherical tolerance zone calculations or ellipsoid tolerance zones.
- Further development of uncertainty calculations.
- Investigation of planar and volumetric geometric arrangements of target points.
- Investigation of the circular tolerance zones where 2D (not 3D) repeatability is useful.
- The addition of thermal compensation.
- Reduced number of test points for efficiency.

CONCLUSIONS

Existing terminology and test methodology for quantifying motion system repeatability remain valid and useful but have the inherent weakness in that they are one-dimensional. The proposed Point Repeatability methodology is the framework for characterizing the repeatability of a functional point in space while accounting for all error sources in three dimensional space. This methodology can be used to simply analyze data collected with existing standard test methods or it can also be used in conjunction with the presented method using a three sensor nest (of capacitive gauges).

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